國立南科國際實驗高級中學
九十七學年度第一次教師甄選
雙語部國中數學科試題卷

第一大題，選擇題，每題 2 分，共 50 分。

1. Use a graphing utility to find any relative minimum or maximum values of the function.
$f(x)=x^3-6x^2+9x+10$

[A] Relative minimum: (3, 15)
Relative maximum: (1, 17)

[B] Relative minimum: (3, 10)
Relative maximum: (1, 14)

[C] Relative minimum: (3, 13)
Relative maximum: (1, 19)

[D] Relative minimum: (1, 14)
Relative maximum: (3, 10)

2. Find the inverse of the function.
$f(x)=\dfrac{-3-3x}{3+4x}$

[A] $f^{-1}(x)=\dfrac{-3x-3}{4x+3}$

[B] $f^{-1}(x)=\dfrac{4x+3}{-3x-3}$

[C] $f^{-1}(x)=\dfrac{-3-3x}{4+3x}$

[D] $f^{-1}(x)=\dfrac{4+3x}{-3-3x}$

3. Divide and write the result in standard form.
$\dfrac{3+2i}{4-3i}$

[A] $\dfrac{18-17i}{25}$ [B] $\dfrac{6+17i}{7}$ [C] $\dfrac{6+17i}{25}$ [D] $\dfrac{18-17i}{7}$

4. Find the vertical asymptote(s), if any, for $f(x)=\dfrac{4x-6}{x^2-11x+30}$.

[A] $x=-7,\ x=5$

[B] $x=5,\ x=6$

[C] $x=5,\ x=6,\ x=-7$

[D] No vertical asymptotes

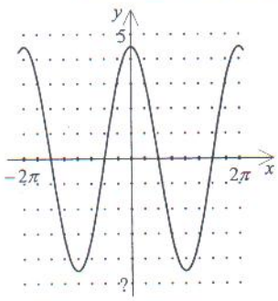
5. Solve the exponential equation algebraically.
$\dfrac{150}{1+e^{-x}}=50$

[A] -0.693 [B] -0.753 [C] -1.386 [D] -0.288

6. On the Richter scale, the magnitude R of an earthquake of intensity I is
$R=\log\dfrac{I}{I_0}$
where $I_0=1$ is the minimum intensity used for comparison. Find the magnitude R of an earthquake if $I=4{,}400{,}000$.

[A] 7.6 [B] 6.6 [C] 6.1 [D] 5.6

7. Find the function shown in the graph.



[A] $y = \cos 4.5x$ [B] $y = 4.5\cos x$ [C] $y = \sin 4.5x$ [D] $y = 4.5\sin x$

8. Evaluate the expression without the aid of a calculator.
arctan 0

[A] 0 [B] 2π [C] $\frac{\pi}{6}$ [D] $\frac{\pi}{4}$

9. Find all solutions of the equation in the interval $[0, 2\pi)$.

$4\sec^2\frac{x}{2} - 7\sec\frac{x}{2} - 2 = 0$

[A] 2π [B] $\frac{2\pi}{3}$ [C] 0 [D] $\frac{4\pi}{3}$

10. Find the exact value of the trigonometric function given that $\sin A = \frac{4}{7}$, $\cos B = -\frac{5}{6}$, $\frac{\pi}{2} \le A \le \pi$, and $\pi \le B \le \frac{3\pi}{2}$.
$\cos(A + B)$

[A] $\frac{1}{42}\left(4\sqrt{11} - 5\sqrt{33}\right)$ [B] $-\frac{11}{42}$

[C] $\frac{1}{42}\left(5\sqrt{33} - 4\sqrt{11}\right)$ [D] $\frac{1}{42}\left(5\sqrt{33} + 4\sqrt{11}\right)$

11. Use the power-reducing formulas to rewrite the expression in terms of the first power of the cosine.
$\tan^2 x$

[A] $\sin x \tan x + 1$ [B] $\frac{1+\cos 2x}{1-\cos 2x}$ [C] $\sin x \tan x - 1$ [D] $\frac{1-\cos 2x}{1+\cos 2x}$

12. Use DeMoivre theorem to find the indicated power of the complex number.
$\left[\sqrt{2}\left(\cos\frac{5\pi}{15} + i\sin\frac{5\pi}{15}\right)\right]^4$

[A] $-2.00 - 3.46i$ [B] $-4.00 + 6.93i$ [C] $-0.71 - 1.22i$ [D] $-0.85 - 3.91i$

13. Find the product, if possible.

AB, if $A = \begin{bmatrix} 0 & 1 & 1 \\ -5 & -1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 3 \\ 0 & 1 \\ -4 & -1 \end{bmatrix}$

[A] $\begin{bmatrix} 0 & -15 \\ 0 & -1 \\ 0 & 0 \end{bmatrix}$ [B] $\begin{bmatrix} -15 & -2 & 1 \\ -5 & -1 & 0 \\ 5 & -3 & 0 \end{bmatrix}$ [C] $\begin{bmatrix} -4 & -5 \\ 0 & -16 \end{bmatrix}$ [D] $\begin{bmatrix} -4 & 0 \\ -5 & -16 \end{bmatrix}$

14. Find the three cofactors of the first row of the matrix.

$$\begin{bmatrix} 6 & -5 & 3 \\ 8 & 7 & -1 \\ -9 & -2 & 4 \end{bmatrix}$$

[A] $C_{11} = ?\ 6,\ C_{12} = ?\ 4,\ C_{13} = 16$ [B] $C_{11} = 26,\ C_{12} = 14,\ C_{13} = ?\ 6$

[C] $C_{11} = 26,\ C_{12} = ?\ 3,\ C_{13} = 47$ [D] $C_{11} = ?\ 6,\ C_{12} = 23,\ C_{13} = ?\ 7$

15. Find the sum using the formulas for the sums of powers of integers.

$$\sum_{n=1}^{15}\left(-\frac{3}{2}+\frac{1}{2}n^2-n\right)$$

[A] -1.5 [B] -6750 [C] 238 [D] 477.5

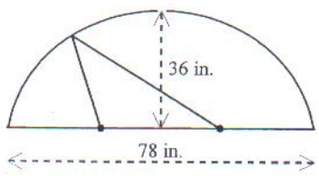
16. A coin is tossed and a die is rolled. What is the probability that the coin shows tails and the die shows a 1 or a 2?

[A] $\frac{1}{12}$ [B] $\frac{5}{6}$ [C] $\frac{1}{4}$ [D] $\frac{1}{6}$

17. Dan is in the chess club. There are 32 students in the club. Three of them will be picked at random to attend an awards banquet. What is the probability that Dan will *not* be randomly chosen to attend the banquet?

[A] $\frac{3}{32}$ [B] $\frac{32}{29}$ [C] $\frac{32}{3}$ [D] $\frac{29}{32}$

18. A fireplace arch is to be constructed in the shape of a semiellipse. The opening is to have a height of 36 inches at the center and a width of 78 inches along the base. To sketch the outline of the fireplace, the contractor uses a 78-inch string tied to two thumbtacks. Where should the thumbtacks be placed?



[A] 21 inches on either side of the center

[B] 15 inches on either side of the center

[C] 13 inches on either side of the center

[D] 20 inches on either side of the center

19. You and a friend live 6 miles apart and are talking on the phone. You hear a crack of thunder and 8 seconds later your friend hears the crack. Find an equation that gives the possible places where the lightning could have occurred. (Use miles as the unit of distance. The speed of sound is about 1100 ft/sec.)

[A] $\frac{x^2}{\frac{25}{36}}+\frac{y^2}{\frac{299}{36}}=1$ [B] $\frac{x^2}{\frac{299}{36}}+\frac{y^2}{\frac{25}{36}}=1$ [C] $\frac{x^2}{\frac{299}{36}}-\frac{y^2}{\frac{25}{36}}=1$ [D] $\frac{x^2}{\frac{25}{36}}-\frac{y^2}{\frac{299}{36}}=1$

20. An asteroid has a parabolic orbit with planet X as its focus. It is 4,000,000 miles away from the center of the planet at its closest point.
(a) Write the polar equation of the orbit of the asteroid with its focus at the pole and its vertex at $\theta = \frac{\pi}{2}$.

(b) Find the distance between the asteroid and the center of planet X when $\theta = -\frac{\pi}{4}$.

[A] (a) $r = \frac{8{,}000{,}000}{1+\sin\theta}$; (b) 27,313,708 miles

[B] (a) $r = \frac{8{,}000{,}000}{1-\cos\theta}$; (b) 27,313,708 miles

[C] (a) $r = \frac{8{,}000{,}000}{1-\cos\theta}$; (b) 59,712,813 miles

[D] (a) $r = \frac{8{,}000{,}000}{1+\sin\theta}$; (b) 59,712,813 miles

21. Find the area of the parallelogram with the given vertices. Is the parallelogram a rectangle?
$A(6, 3, 1)$, $B(1, 5, 0)$, $C(1, 3, 6)$, $D(6, 1, 7)$

[A] 73.24; Yes [B] 73.24; No [C] 33.17; No [D] 33.17; Yes

22. Find the volume of the parallelepiped that has the adjacent edges
$\mathrm{u} = -7\mathbf{i}+8\mathbf{j}+9\mathbf{k}$, $\mathrm{v} = -3\mathbf{i}+4\mathbf{j}$, and $\mathrm{w} = -3\mathbf{i}+2\mathbf{j}+2\mathbf{k}$.

[A] 46 [B] 1 [C] 108 [D] 2002

23. Let $g(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ for $f(x) = \frac{1}{-2x^5-4}$ Find $g(0)$.

[A] ∞ [B] $-\infty$ [C] 0 [D] 0.125

24. Find the limit.
$$\lim_{x\to\infty}\left(\frac{2x^3}{x^2-4}+\frac{5x}{x+4}\right)$$

[A] 7 [B] 2 [C] ∞ [D] 10 [E] None of these

25. Evaluate the summation $S = \sum_{i=1}^{n}\frac{i+2}{n^2}$ for $n = 10^2$. What is the limit of S as $n \to \infty$?

[A] $\frac{21}{20}; \frac{1}{2}$ [B] $\frac{21}{40}; 1$ [C] $\frac{21}{40}; \frac{1}{2}$ [D] $\frac{21}{20}; 1$

Answer

1. BACBA 6. BBABD

11. DADCD 16.DDBDA

21.CACCC

雙語部國中數學

第二大題、計算題（1-5 題每題 8 分，6-10 每題 2 分）

1、A ship leaves port at noon and heads due west at 20 knots, or 20 nautical miles (nm) per hour. At 2 P.M. the ship change course to N 60° W. Find the ship's bearing and distance from the port of departure at 3 P.M.

2、Find the standard form of the equation of hyperbola with foci $(-1,2)$ and $(5,2)$ and vertices $(0,2)$ and $(4,2)$. And state asymptotes of the hyperbola.

3、The plane E passes through the point $(2,2,1)$ and $(-1,1,-1)$ and is perpendicular to the plane $2x-3y+z=3$. Find the general form of the equation of the plane E. point $P(1,1,1)$ ，find the distance of P and E.

4、Graph the system of inequalities $\begin{cases} y > -x^2 - 4x \\ y \le x^2 + 7x + 10 \end{cases}$

5、Graph the function $f(x)=(x+3)(x-1)(x-4)$

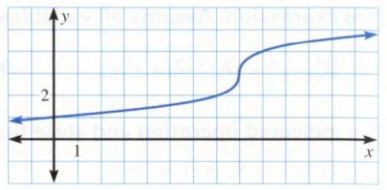
6. What are the value of x and y in the matrix equation $2\begin{bmatrix} -2 & -1 \\ -10 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} -16 & -8 \\ 4 & 40 \end{bmatrix}$

（A）$x=8, y=-80$（B）$x=-4, y=-80$（C）$x=4, y=80$（D）$x=4, y=-80$（E）$x=2, y=-40$

7、If x^2+8x+c is a perfect square trinomial, what is the value of c

（A）4（B）8（C）16（D）32（E）64

8、Which function is graphed？（A）$y=\sqrt[3]{x-3}+8$



（B）$y=\sqrt[3]{x+3}+8$（C）$y=\sqrt[3]{x-8}+3$

（D）$y=\sqrt[3]{x+8}-3$（E）$y=\sqrt[3]{x+8}+3$

9、What is the solution of the equation $0.5\log_3 x=2$？（A）4（B）64（C）81（D）$\frac{1}{64}$（E）$\frac{1}{81}$

10、Which function is the inverse of $y=\ln(x-2)$？（A）$y=e^x+2$（B）$y=e^{x+2}$（C）$y=e^x-2$（D）$y=e^{x-2}$

（E）$y=e^{-2x}$

第二大題答案:

1. distance $20\sqrt{7}$ ， the angle with the north-south is $90°-\tan^{-1}\frac{\sqrt{3}}{5}$

2. $\frac{(x-2)^2}{4}-\frac{(y-2)^2}{5}=1$ ，asymptotes $\frac{x-2}{2}\pm\frac{y-2}{\sqrt{5}}=0$

3. $7x+y-11z+5=0, \frac{8}{\sqrt{171}}$

6- 10　DCCCA